Nonlinear magnetodynamics of a single-domain particle: the high-barrier approximation

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Abstract

Micromagnetic Fokker–Planck equation with allowance for the gyration term is solved for a uniaxial particle in the low-temperature limit. The solution is obtained in the form of an asymptotic series in the parameter that is the barrier height-to-temperature ratio, and in these terms the superparamagnetic relaxation time and linear and cubic dynamic susceptibilities are presented. The effect of strong enhancement of cubic susceptibility in the weak precession damping limit, formerly found in numerical simulations, is proven analytically.

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Nowadays the interest to superparamagnetic relaxation is mainly due to the search of means of enhancing the magnetic recording density. Whereas the linear response theory for magnetic nanoparticles is a well-developed field, its nonlinear counterpart until recently comprised of a few fragments. In Refs. [1,2] we have started to consistently treat the dynamic nonlinear susceptibilities of a single-domain particle and of orientationally textured dilute assemblies of the latter. In particular, we build up an analytical approach specialized for the case where the anisotropy potential barrier \( \Delta U_a \) is greater than the thermal energy \( kT \). However, our model retains the effects of both fast and slow “interwell” motions of the particle magnetic moment. In this high-barrier limit all the results are expressed as asymptotic series in inverse powers of \( s = \frac{\Delta U_a}{kT} \).

Hereby we extend the consideration [2] from a purely relaxational case, for which it had been built originally, to the full magnetodynamical framework, i.e., include the gyromagnetic effect. To test the theory, we apply it to the case of a nonlinear effect discovered numerically in Ref. [3]. There the numeric simulation showed that in a uniaxial superparamagnetic particle the cubic magnetic susceptibility \( \chi^3 \) is sensitive to the spin–lattice relaxation rate rendered by the damping constant \( z \) in the Landau–Lifshitz equation. Namely, for \( z \ll 1 \) (the Larmor precession with a high quality factor) both \( \Re \) and \( \Im \) of \( \chi^3 \), retaining their shapes as the functions of frequency, grow in magnitude close to \( \propto \frac{1}{z} \). Remarkably, this occurs at \( \omega \tau_D \ll 1 \), i.e., well below the usual FMR range; here \( \tau_D \) is the magnetic rotary diffusion time.

The compact form of the full micromagnetic Fokker–Planck equation is [5]

\[
2\tau_D \frac{\partial W}{\partial t} = \mathbf{J} W \mathbf{Q}(U/kT + \ln W),
\]

where \( W(\mathbf{e}, t) \) is the orientational distribution function of the magnetic moment \( \mathbf{u} = \mu \mathbf{e} \), \( \mathbf{J} = (\mathbf{e} \times \partial/\partial \mathbf{e}) \) the infinitesimal rotation operator with respect to \( \mathbf{e} \), and \( \mathbf{Q} = \mathbf{J} + (1/2z)\partial/\partial \mathbf{e} \). For the orientation-dependent magnetic energy of a single-domain particle in the field \( \mathbf{H} = H\mathbf{n} \) we take

\[
U = -\mu H(\mathbf{e} \cdot \mathbf{h}) - \Delta U_a(\mathbf{e} \cdot \mathbf{n})^2
\]

with \( \mathbf{n} \) being a unit vector of the easy axis. The low-frequency magnetodynamics of such a particle is...
dominated by the interwell mode $W_1(\xi)$, its eigenvalue writes $\lambda_1 = 2/\tau_1$, where the relaxation time is

$$\tau_1 = \tau_0 \frac{\sqrt{\pi e^\sigma}}{2\sigma^{3/2}} \left( 1 + \frac{1}{\sigma} + \frac{7}{4\sigma^2} + \frac{9}{2\sigma^3} + \cdots \right).$$

(3)

Exponential dependence of $\lambda_1$ on the barrier height $\sigma$ means that the eigenfunction $W_1$ is a solution of a Kramers-type problem. Accordingly, we set the left-hand side of Eq. (1) to zero and find $W_1$ from the residual equation with exponential accuracy. (Similar approach was used in Ref. [4] to study the escape rate of a classical spin with the Hamiltonian (2)). The essential feature of the problem is that Eq. (1) with the zero left-hand part has also another solution, an exact one. Indeed, the equilibrium (Boltzmann) function $N_0$ sets the inner parenthesis to identical zero. At low frequencies $W_0$ accounts (in the adiabatic limit $\omega \tau_{w,2,3,\ldots} = 0$) for the effect of all the intrawell modes. Using $W_0$ and $W_1$ (see Ref. [2] for the details), we get the linear and cubic susceptibilities of the system in the form

$$\chi^{(1)}_{\omega \sigma} = \chi_0^{(1)} \left( B_0^{(1)} + \frac{B_1^{(1)}}{1 + i\sigma \tau_1} \right),$$

$$\chi^{(3)}_{\omega \sigma} = \frac{1}{4} \chi_0^{(3)} \left( B_0^{(3)} + \frac{B_1^{(3)}}{1 + i\sigma \tau_1} + \frac{B_2^{(3)}}{1 + 3i\sigma \tau_1} \right),$$

(5)

with the static susceptibilities

$$\chi_0^{(1)} = c\mu^2/kT, \quad \chi_0^{(3)} = c\mu^4/(kT)^3,$$

here $c$ is the particle number concentration and the amplitudes $B_k^{(1)}$ are evaluated by expanding the functions $W_0$ and $W_1$ with respect to $\xi = \mu H/kT$.

With allowance for the gyration term in Eq. (1), function $W_1$ acquires dependence on the damping parameter $\alpha$. This results in modification of the coefficients $B_k^{(1)}$ and $B_k^{(3)}$ in Eq. (5). Then for a randomly oriented particle assembly one gets $\chi^{(3)}$ in the form of Eq. (5) with $\tau_1$ from Eq. (3) and

$$B_0^{(3)} = \frac{1}{30\sigma^3} + \frac{47}{240\sigma^4} + \frac{49}{40\sigma^5} + \cdots,$$

$$B_1^{(3)} = \frac{1}{15} - \frac{1}{6\sigma} - \frac{11}{120\sigma^2} + \cdots + F(\sigma, \alpha),$$

$$B_3^{(3)} = -\frac{2}{15} + \frac{3}{10\sigma} + \frac{7}{120\sigma^2} + \cdots - F(\sigma, \alpha).$$

(6)

Function $F(\sigma, \alpha)$ at large $\alpha$ reduces to $F = -1/240\sigma^2 + \cdots$ in full agreement with the high-damping limit considered in Ref. [2]. At small $\alpha$ it behaves as $1/\alpha$, see Fig. 1, causing hyperbolic growth of $\chi^{(3)}$.

In conclusion: we have found that analytical formulas (5) and (6) are in fairly good agreement with the numerical results of Ref. [3].

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References