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Spectral Properties of Helical Turbulence

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Abstract—The spectral properties of homogeneous stationary turbulence excited by a force that introduces considerable helicity, along with energy, into the flow are considered. It is shown that helicity is transferred on the inertial range as a passive admixture and its spectrum obeys the "$-5/3$" law much more accurately than the velocity fluctuation spectrum. The helicity is dissipated on the same scales as the energy, though the helicity transfer dynamics on the inertial range are different on the large and small scales. Numerical experiments are performed on the basis of a cascade model developed for describing helical turbulence.

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Interest in helical turbulence first arose in connection with the problem of magnetic field excitation by electrically conducting fluid flows [1] in which the loss of mirror symmetry on the scale of the small-scale fluctuations makes it possible to obviate the so-called exclusion theorems (antidynamo theorems) which impose rigid constraints on the topology of the mean velocity fields capable of generating large-scale magnetic fields. The helicity (to be more precise, helicity density) is defined as an integral of the scalar product of the velocity and vorticity and is an integral of motion in three-dimensional ideal incompressible fluid dynamics. The concept of helical turbulence is widely used in astrophysical and, to a lesser degree, geophysical fluid dynamics, although so far the question of the effect of the helicity on the turbulent flow dynamics has remained unanswered.

1. BASIC NOTIONS AND FORMULATION OF THE PROBLEM

The helicity $H = V^{-1}\int_V(\mathbf{u}\,\mathrm{curl}\,\mathbf{u})\,dV$ is the integral of motion with an arbitrary sign; for the spectral density of the helicity $H(k)$ it is possible only to indicate the upper bound

$$|H(k)| \leq kE(k). \tag{1.1}$$

Here, $E(k)$ is the spectral density of the velocity fluctuation energy and k is the wavenumber. Precisely for this reason, the second integral of motion does not lead to such rigorous consequences as in two-dimensional turbulence in which two positively defined quantities (energy and enstrophy), which conserve their values, can be transferred only toward opposite ends of the spectrum. Condition (1.1) leaves room for two scenarios of the behavior of the helicity and its influence on the dynamics of a three-dimensional turbulent flow [2]. First, it can be assumed that a direct helicity cascade toward small scales is realized and is accompanied by an inverse energy cascade. No corroboration of this scenario has yet been obtained. Second, it is not improbable that the energy and the helicity are simultaneously transferred to small scales, the helicity being transferred as a passive admixture. This scenario is confirmed by the many numerical experiments carried out in [3], as well as experimental investigations [4].

According to the Kolmogorov concept of developed turbulence, the statistical properties of homogeneous, isotropic, stationarily excited turbulence on the inertial range are fully determined by the energy dissipation rate ε, equal to the specific power of the external forces. Helical turbulence involves a second important parameter, namely, the helicity dissipation rate η equal to the helicity inflow from the external sources per unit time per unit fluid mass. However, the question of the effect of the helicity on the cascade is still poorly understood. In [5] it was inferred that the helicity is dissipated on a scale greater than the energy dissipation scale, that is, the inertial range of helicity transfer is shorter than the inertial range of energy transfer. On the contrary, in [6] it is argued that a constant helicity flux is established along the entire inertial range, while the spectral density of the helicity is governed in this case by the law $H(k) \sim k^{-5/3}$. This means that the helicity is transferred over the spectrum as a passive admixture. In [7] the known deviation from the Kolmogorov "$-5/3$" law is attributed to the effect of helicity rather than intermittency, which leads to an $E(k) \sim \varepsilon^{0.63}\eta^{0.03}k^{-1.7}$ law. In [8] it was assumed that the helicity transfer processes can become important precisely on small scales leading to the appearance of an $E(k) \sim k^{-4/3}$ range, adjacent to the dissipation range, in the energy spectrum. The results of direct numerical simulation of the helical turbulence are presented in [6, 8]; however, the range of Reynolds number values reached did not make it possible to obtain an inertial range sufficient for interpreting the results with any assurance.

An effective tool for studying cascade processes in developed homogeneous turbulence is furnished by cascade models. These models describe the spectral transfer processes by means of a few variables U_n, each of which is a collective characteristic of the velocity field fluctuation amplitudes on the wavenumber range $k_n < |k| < k_{n+1}$, where $k_n = \beta^n$ and β is the range width (shell thickness). The equations for U_n must reproduce the basic properties of the Navier–Stokes equations, namely, have the same integrals of motion and the same form of nonlinearity. Cascade models have been used since the seventies of the last century [9–13]; however, their particularly active investigation started in the nineties [14–17] after it had been shown that they reproduce the behavior of the higher-order structure functions in actual turbulence [18]. At that time the now universally accepted term "shell models" first appeared. The cost of their simplicity is the complete absence from these models of information on the spatial distribution of the turbulent flow characteristics, which limits their range of employment to problems in which it is permissible to use the concept of homogeneous, or locally-homogeneous, turbulence. A detailed description of the principles of construction of cascade models and their possibilities can be found in [19, 20].

The first and simplest cascade turbulence model is the Novikov–Desnyanskii model [9] introduced for real variables in the form:

$$\dot{U}_n = k(U_{n-1}^2 - bU_nU_{n+1}) - \nu k^2 U_n. \tag{1.2}$$

A model of form (1.2) can at any one time ensure the conservation of only one integral of motion, whose form is determined by the parameter b. In the dissipationless limit the requirement of energy $E = \sum U_n^2/2$ conservation leads to $b = \beta$, whereas the requirement of enstrophy conservation leads to $b = \beta^3$. In this model it is impossible to describe helicity.

The model that has enjoyed the widest application is the so-called GOY model [10, 18], which in complex variables can be written in the form:

$$\dot{U}_n = ik_n\left(U_{n+1}^*U_{n+2}^* - \frac{\zeta}{2}U_{n-1}^*U_{n+1}^* + \frac{\zeta - 1}{4}U_{n-2}^*U_{n-1}^*\right) - \nu k_n^2 U_n^* \tag{1.3}$$

and its modification known as the SABRA model [21], which differs with respect to the mode of complex conjugation of the terms on the right side of Eq. (1.3). Models of this type are constructed on the basis of the interaction between three neighboring shells (in model (1.2) it is the modes belonging to only two shells that interact) and have two integrals of motion. The first is the energy integral, while the second depends on the parameter ζ, this integral being also positive definite for $\zeta > 1$ (at $\zeta = 5/4$ the dimensionality of this integral is the same as that of the enstrophy), while for $\zeta < 1$ it is the quantity $W = \sum(-1)^n k_n^\mu |U_n|^2$ of arbitrary sign that is conserved; here, $\mu = -\log_\beta|\zeta - 1|$. At $\zeta = 1/2$ two quadratic quantities are conserved:

$$E = \frac{1}{2}\sum|U_n|^2, \qquad W = H = \sum(-1)^n k_n|U_n|^2. \tag{1.4}$$

The first quantity is the energy, while the dimensionality of the second coincides with that of the hydrodynamic helicity; and precisely at this value of the parameter ζ the model closely reproduces the statistical properties of developed three-dimensional turbulence, which has attracted the widespread attention of researchers. Model (1.3) has been generalized to cover the case of magnetohydrodynamic turbulence [22] and used for describing the role played by small-scale helical turbulence in the problem of magnetic field generation by turbulent electrically-conducting fluid flows [23].

However, models of type (1.3) have a fundamental shortcoming related with the fact that in them the helicity is strictly proportional to the shell energy, while its sign is determined by the evenness of the shell number. Thus, we arrive at a nonphysical situation in which the presence of fluctuation energy on a certain wavelength range necessarily implies the presence of helicity of a prescribed sign. The appearance of noticeable mean helicity leads to the situation in which the energy spectrum takes a characteristic saw-tooth form and the growth of helicity results in the blocking of the cascade transfer mechanism. The GOY model has been used for modeling helical turbulence, for example, in [5] but, as distinct from DNS [6], gave a much more homogeneous spectral flux of helicity than of energy. An attempt to construct a model for describing cascade processes in helical turbulence was made in [24], where a second variable, describing the fluctuation energy with helicity opposite in sign, was introduced for each shell. This led to a lack of uniqueness of the model thus constructed and, ultimately, provided no means of obtaining conclusive results.

Another approach consists in determining the helicity for cascade models by means different from that Eq. (1.4). It involves a quadratic quantity with alternating signs, which can be expressed in terms of the variables of a given shell and has the dimensionality of helicity. For complex variables $U_n = a_n + ib_n = u_n \exp(i\phi_n)$ as the helicity we can use the quantity

$$H = \frac{i}{4}\sum((U_n^*)^2 - U_n^2) = \sum k_n a_n b_n, \tag{1.5}$$

which characterizes the degree of correlation between the real and imaginary parts of the variables. As shown in [25], the dynamics of models of type (1.3) with helicity (1.5) differ considerably from the expected behavior and do not lead to the well-known results for Kolmogorov nonhelical turbulence. At the same time, already in [26] it was shown that an integral of form (1.5) appears in a very simple model of type (1.2), if written in complex variables. This model was used in [27] to analyze the statistical characteristics of freely degenerating turbulence. It is important to note that this model admits solutions corresponding to strictly nonhelical motions ($H \equiv 0$), which is excluded in models of the GOY type (1.3).

The purpose of this study is to consider the cascade model for helical turbulence and to employ it for investigating the distinctive features of cascade helicity transfer in stationary homogeneous turbulence with a constant helicity supply.

2. CASCADE MODEL

The general form of the equations for the complex variables U_n based on the interactions of only two neighboring shells and satisfying the law of conservation of energy and the law of conservation of helicity in the form (1.5) is as follows:

$$\begin{aligned}\dot{U}_n = ik_n\gamma_1\big[U_{n-1}^2 + (U_{n-1}^*)^2 + \beta(U_n^*U_{n+1} - U_n^*U_{n+1}^*)\\ - \beta^2(U_nU_{n+1} - U_nU_{n+1}^*)\big] + ik\gamma_2\big[U_{n-1}U_n + U_{n-1}^*U_n + \beta(U_{n-1}^*U_n^* - U_{n-1}U_n^*)\\ - \beta^2(U_{n+1}^2 + (U_{n+1}^*)^2\big] - \mathrm{Re}^{-1}k^2U_n + F_n.\end{aligned} \tag{2.1}$$

The first part of the nonlinear term is the generalization of Eq. (1.2) to the case of complex variables and describes the interactions of a given shell with the previous one. Its second part symmetrically describes the interaction of the given shell with the next one. The coefficients γ_1 and γ_2 determine the weights of these parts. In the calculations we used the values $\gamma_1 = 1$ and $\gamma_2 = -\beta^{-5/2}$ proposed in [26] on the basis of an estimate of the number of interacting vortices of different dimensions.

The term $F_n = f_n \exp(i\theta_n)$ describes the action of the external forces that maintain the energy and helicity inflow on the integral scale of turbulence. The forces act only on the largest scales ($n = 0$ and 1 shells) and ensure fixed inflows of energy and helicity (ε and η, respectively) into the system. The force amplitudes are as follows:

$$f_0 = -\frac{\varepsilon k_1 \sin(\theta_1 + \phi_1) + \eta\cos(\theta_1 - \phi_1)}{u_0 z},$$
$$f_1 = -\frac{\varepsilon k_0 \sin(\theta_0 + \phi_0) + \eta\cos(\theta_0 - \phi_0)}{u_1 z}, \tag{2.2}$$
$$z = k_0\cos(\theta_1 - \phi_1)\sin(\theta_0 + \phi_0) - k_1\cos(\theta_0 - \phi_0)\sin(\theta_1 + \phi_1),$$

while the phases θ_1 and θ_2 take random values which vary over a given time interval representing the correlation time for an external random force. In terms of model (2.1) the most important characteristics of the inertial range, i.e., the spectral energy and helicity fluxes on the given scale, are determined by the formulas

$$\Pi_n = \mathrm{Im}\left\{k_n\left[\beta\gamma_1\left(U_n^2 + (U_n^*)^2\right)\left(U_{n+1} - U_{n+1}^*\right)\right] + \left[\beta^2\gamma_2\left(U_{n+1}^2 + (U_{n+1}^*)^2\right)\left(U_n - U_n^*\right)\right]\right\},$$
$$\Xi_n = \mathrm{Im}\left\{k_n^2\left[\beta^2\gamma_1\left(U_n^2 + (U_n^*)^2\right)\left(U_{n+1} + U_{n+1}^*\right)\right] + \left[\beta^2\gamma_2\left(U_{n+1}^2 + (U_{n+1}^*)^2\right)\left(U_n + U_n^*\right)\right]\right\}.$$

We will consider forced turbulence maintained by a force of form (2.2) with a fixed intensity ($\varepsilon = 1$) and different levels of the helicity introduced into the flow $0 \le \eta \le 1$. The calculations are performed on the Reynolds number range $10^3 \le \mathrm{Re} \le 10^6$. The fact that the Reynolds numbers are relatively low for cascade models is due to the desire to investigate a situation accessible for DNS. In the model under consideration it is the value $\mathrm{Re} = 3 \times 10^3$ that corresponds to the most detailed DNS of helical turbulence presented in [3]. As the correspondence criterion we will take the inertial range length determined from the constancy of the spectral energy flux. In all calculations the shell thickness was taken to be $\beta = 1.618$.

3. HELICITY CASCADE

It is the question of the length and structure of the inertial range of helicity transfer that gives rise to most discussion. In [5] it was hypothesized that helicity dissipation starts on a scale λ_H different from the Kolmogorov energy dissipation scale $\lambda_E \approx (\nu^3/\varepsilon)^{1/4}$. The latter can be derived from the assumption that on any scale l the energy dissipation rate can be estimated as $\nu u_l^2/l^2$, while for $l = \lambda_E$ it becomes equal to the power supplied ε. In [5] it was assumed that, by analogy, for the helicity dissipation rate we can take

$$\frac{dH_l}{dt} \approx \nu\frac{u_l\omega_l}{l^2} \approx \nu\frac{u_l^2}{l^3}, \tag{3.1}$$

where ω is a characteristic vorticity on the given scale l, and that there must be a scale λ_H on which this quantity becomes equal to the helicity supplied:

$$\nu\frac{u_\lambda^2}{\lambda_H^3} \approx \eta, \qquad \lambda_H \approx \left(\frac{\nu u_\lambda^2}{\eta}\right)^{1/3} \sim \nu^{3/7}\eta^{-3/7}\varepsilon^{2/7}.$$

In the above formula the Kolmogorov estimate $u_l \approx (\varepsilon l)^{1/3}$ was used. The main conclusion reached in [5] was that the ratio of the two dissipative scales increases with Re (with reduction in viscosity) as

$$\frac{\lambda_E}{\lambda_H} \approx \nu^{9/28}. \tag{3.2}$$

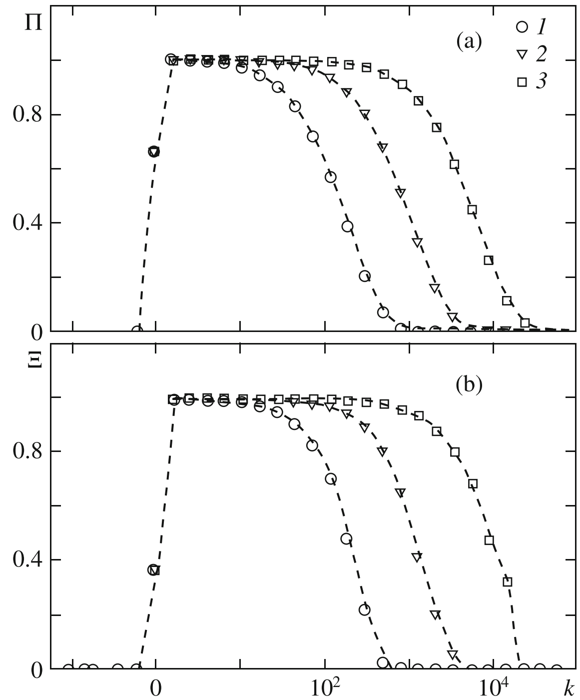

Fig. 1. Spectral fluxes of energy (a) and helicity (b) at different Reynolds numbers; (*1*) to (*3*) correspond to $\mathrm{Re} = 3 \times 10^3$, 3×10^4, and 3×10^5.

An alternative point of view is that energy and helicity dissipation occurs on the same scales [3, 6]. The concept of the coincidence of the dissipative scales $\lambda_H \approx \lambda_E$ is taken as a postulate and it is assumed that over the entire inertial range the spectral distributions of energy and helicity established are similar (accurate to corrections due to intermittency) to the "$-5/3$" law

$$E(k) \sim \varepsilon^{2/3} k^{-5/3}, \qquad H(k) \sim \eta \varepsilon^{-1/3} k^{-5/3}. \tag{3.3}$$

In [5] this inference was confirmed by the DNS results obtained for relatively small Re values.

As for the conclusion contained in Eq. (3.2) that in developed turbulence helicity is dissipated on considerably larger scales than energy, it is based on the incorrect estimate (3.1), which ignores the fact that the helicity dissipation (like the helicity itself) cannot be estimated from the product of the characteristic values of the velocity and the vorticity. The helicity ensures only the correlation of the velocity and vorticity fields, while estimate (3.1) gives a finite helicity dissipation rate in nonhelical turbulence as well. Thus, instead of Eq. (3.1), we should write

$$\frac{dH_l}{dt} \approx \nu \frac{\langle u_l \omega_l \rangle}{l^2} \approx \nu \frac{C_l u_l^2}{l^3}, \qquad C_l = \frac{\langle u_l \omega_l \rangle}{\left(\langle u_l^2 \rangle \langle \omega_l^2 \rangle\right)^{1/2}}, \tag{3.4}$$

where C_l is the coefficient of correlation of the velocity and vorticity fluctuations on the corresponding scale l. Using estimate (3.4) instead of (3.1) leads to the situation in which the dissipative helicity scale depends on the correlation coefficient, which itself depends on the scale l

$$\lambda_H \sim \left(\frac{\nu}{\eta} C_\lambda\right)^{3/7} \varepsilon^{2/7}.$$

It should be noted that the co-existence of spectra (3.3) implies a linear dependence of the correlation coefficient on the scale l:

$$C_l \approx \frac{H_l l}{E_l} \sim \frac{\eta}{\varepsilon} l, \tag{3.5}$$

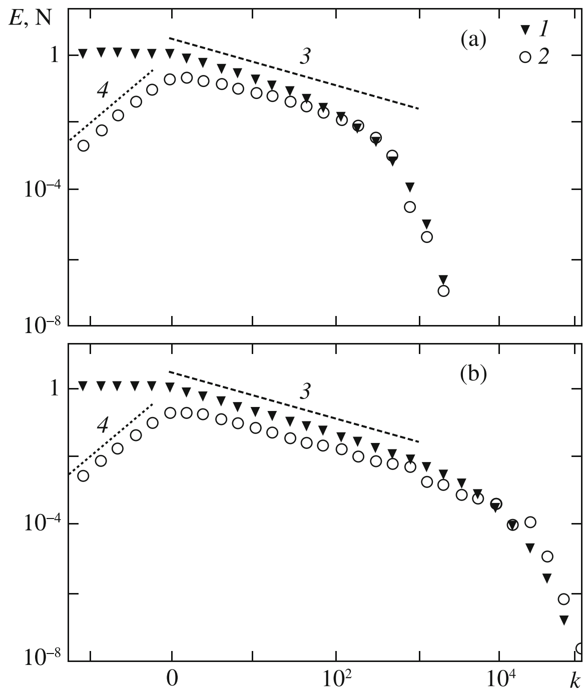

Fig. 2. Distributions of the mean values of the energy (*1*) and the absolute value of helicity (*2*) in forced turbulence; (a) and (b) correspond to $\mathrm{Re} = 3 \times 10^3$ and 3×10^5; (*3*) and (*4*) relate to $E(k) \sim k^{-5/3}$ and $E(k) \sim k$.

which on the dissipation scale ($\lambda_H \approx \lambda_E$) gives the expression

$$C_\lambda \approx \frac{\eta}{\varepsilon^{5/4}} \nu^{3/4}. \tag{3.6}$$

Formula (3.6) agrees with the estimate for the mean angle between the velocity and vorticity vectors on the dissipative scale [28] derived from the asymptotic model.

4. NUMERICAL EXPERIMENTS

The calculations performed within the framework of the cascade model (2.1) showed that for all the Reynolds numbers considered the scales reached by the energy and helicity fluxes are actually similar in value, that is, $\lambda_H \approx \lambda_E$. The inertial range (Fig. 1) determined from the constancy of the energy flux starts to form at $\mathrm{Re} = 10^3$ and extends approximately to three decades at $\mathrm{Re} = 10^6$. Qualitatively, the helicity flux curves are of the same form as the energy flux curves but obtaining a smooth curve for the helicity flux requires averaging over considerably greater time intervals. Whereas a smooth energy flux is obtained by averaging the solutions over time intervals of several tens of dimensionless time units, the helicity fluxes shown in Fig. 1b required integration over the intervals $10^2 < t < 10^3$ for $\mathrm{Re} = 3 \times 10^3$ and $10^3 < t < 10^4$ for $\mathrm{Re} = 3 \times 10^5$.

The time-averaged energy and helicity distributions obtained by numerically integrating the cascade equations for different Reynolds numbers are presented in Fig. 2. The first plot (Fig. 2a) was obtained for $\mathrm{Re} = 3 \times 10^3$. As noted above, this case corresponds most closely to the calculations [3] and, in fact, gives proportional energy and helicity distributions on the $2 < k < 10$ range. In this case, the spectrum slope is somewhat greater than the usual values for developed turbulence (-1.77 rather than the usual -1.71) which can be attributed to an insufficiently high Reynolds number. The calculations also involved the infrared spectrum region, that is, scales greater than the scale of action of the external forces ($k < 1$). On these scales the shell energies are uniformly distributed ($E_n \sim$ const, that is, $E(k) \sim k^{-1}$) but helicity is transferred toward the larger scales much less efficiently, thus giving the distribution $H_n \sim k_n^2$, which corresponds to an $H(k) \sim k$ law.

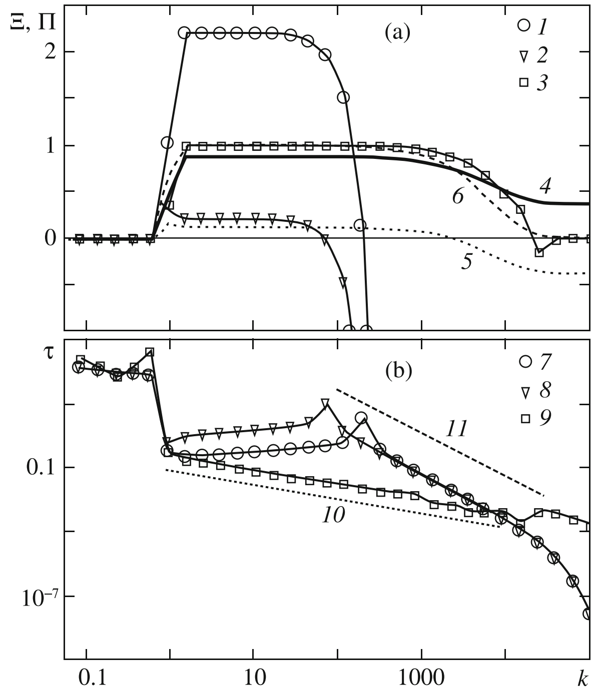

Fig. 3. Separated helicity and energy fluxes (a): (*1*) Ξ^+; (*2*) Ξ^-; (*3*) Ξ; (*4*) Π^+; (*5*) Π^-; and (*6*) Π; and transfer times (b): (*7*) τ^+; (*8*) τ^-; and (*9*) τ; (*10*) and (*11*) relate to $k^{5/3}$ and $k^{-7/3}$ for $\mathrm{Re} = 3 \times 10^5$.

With increase in Re the spectral energy distribution demonstrates an increasingly clearly expressed inertial range with the law $E(k) \sim k^{-1.71}$ (Fig. 2b). It is important to note that on this range the helicity follows a "$-5/3$" law fairly accurately for all three Reynolds numbers, whereas the energy spectrum slope varies. This confirms that the helicity behaves as a passive admixture, since it is known that for passive scalar admixtures the spectral density of the fluctuation intensity obeys the power-law distributions obtained from dimensional analysis much better than the spectral density of the velocity fluctuation energy [20]. On smaller scales (greater k) the mean helicity distribution becomes increasingly "noisy" and gives smooth dependences only for very long averaging times (the helicity distribution in Fig. 2b was obtained by averaging over 50 realizations, each 10^4 dimensionless time units in length).

The distinctive features of the spectral transfer in helical turbulence can be demonstrated by expanding the energy in components related with a certain sign of the helicity. The cascade variables make it possible to perform this expansion in exactly the same way as for the complete fluid dynamics equations, all symmetry properties being conserved. By analogy with [6], in terms of the cascade variables the following definitions can be introduced:

$$\frac{H_n^+}{k_n} = E_n^+ = \frac{U_n^2}{4}, \qquad \frac{H_n^-}{k_n} = E_n^- = \frac{(U_n^*)^2}{4},$$

from which there follows

$$E_n = E_n^+ + E_n^-, \qquad H_n = H_n^+ - H_n^-.$$

The spectral fluxes of energy and helicity can also be divided into two parts ($\Pi_n^\pm$ and $\Xi_n^\pm$). As shown in [6], the fluxes of the positive Ξ^+ and negative Ξ^- helicity components increase with the wavenumber k. The conserved quantity, that is, their difference, or the total flux, is constant over the entire inertial range.

The separation of the energy $\Pi^\pm$ and helicity $\Xi^\pm$ fluxes (Fig. 3a) shows that in this case the main energy flux is accounted for by Π^+ whose behavior is only slightly different from that of the total flux. The helicity flux Ξ^+ also predominates on large scales but the range on which the separate fluxes Ξ^+ and Ξ^- are constant turns out to be considerably shorter than the inertial range. Clearly, on the scales $80 \le k \le 100$ a constant helicity flux Ξ is conserved against the background of rapidly growing absolute values of the negative fluxes $\Xi^+$and Ξ^-. The conclusion that on injection of positive helicity the flux from positive to negative modes

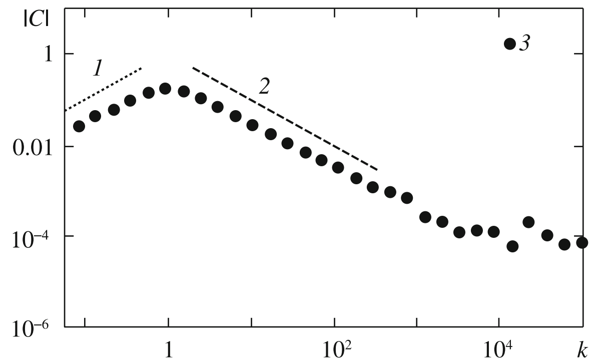

Fig. 4. Coefficient of correlation between the velocity and vorticity fluctuations for a given scale: $C(k) \sim k$ (*1*); $C(k) \sim k^{-1}$ (*2*); and results of calculations (*3*).

predominates on the inertial range and that on small scales the dissipation mainly occurs precisely in the negative modes was reached in [6].

The spectral helicity distributions presented in Fig. 2 confirm the tendency toward sign reversal in the dissipative scales. It is important to note that in the small-scale region of the inertial range the total positive helicity flux is determined by the dominant inverse flux of negative helicity. The different dynamics of the helicity flux on the large and small scales are also illustrated by Fig. 3b, which shows the characteristic helicity transfer times determined for the total and separate fluxes as the ratios

$$\tau_n = \left|\frac{H_n}{\Xi_n}\right|, \qquad \tau_n^{\pm} = \left|\frac{H_n^{\pm}}{\Xi_n^{\pm}}\right|.$$

Since the helicity obeys the power law $H_n \sim k_n^{-2/3}$ and the total helicity flux is constant, the transfer time obeys the same power law over the entire inertial range. For the separate fluxes the times $\tau_n^{\pm}$ have two clearly expressed ranges with different behavior, separated by the point of sign change of the corresponding flux (the negative helicity flux changes sign earlier than the positive helicity flux). To the left of this boundary, that is, in the large-scale region of the inertial range, the transfer times gradually increase (the transfer efficiency decreases) and $\tau_n^+ < \tau_n^-$, since it is the direct flux of positive helicity that predominates. In the small-scale region of the inertial range the times τ_n^+ and τ_n^- become closer in value and rapidly decrease with the scale $\tau_n^{\pm} \sim k_n^{-7/3}$. This can be interpreted as an inverse flux of negative helicity generated on the dissipative scale, whose intensity decreases as the scale increases.

Figure 4 confirms the conclusion concerning the scale dependence of the correlation coefficient. Clearly, this coefficient does in effect decrease over the entire inertial range in accordance with a law similar to (3.5).

Summary. It is shown that for a constant helicity supply on the turbulence excitation scale the helicity is transferred over the entire inertial range as a passive admixture and is dissipated on the same scales as the energy. In this case, the coefficient of correlation between the velocity and helicity fluctuations decreases in proportion to the fluctuation scale. A cascade turbulence model, equally effective for both helical and nonhelical turbulence, is developed. The calculations performed on the basis of this model show that at high Reynolds numbers within the helical turbulence an inertial range is formed, with a spectral energy distribution usual for developed turbulence and differing from the "$-5/3$" law due to intermittency. At the same time, a stationary helicity flux is observable over the entire inertial range, with the spectral helicity density closely obeying the "$-5/3$" law.

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