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Measurements of Turbulent Magnetic Diffusivity in a Liquid-Gallium Flow

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Direct measurements of the effective conductivity (magnetic diffusivity) in the turbulent flow of a liquid metal have been performed. An nonstationary turbulent flow of a gallium alloy has been excited in a closed toroidal channel with dielectric walls. The Reynolds number reaches a maximum value of $Re \approx 10^6$, which corresponds to the magnetic Reynolds number $Rm \approx 1$. The conductivity of the metal in the channel has been determined from the phase shift of forced harmonic oscillations in a series RLC circuit whose inductance is a toroidal coil wound around the channel. The maximum deviation of the effective conductivity of the turbulent medium from the ohmic conductivity of the metal is about 1%.

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1. INTRODUCTION

The most important fundamental problem of magnetic hydrodynamics is the problem of the generation of cosmic magnetic fields. These fields are generated by flows of a conductive medium in the interiors of planets, the convective shells of stars, or galactic discs and are characterized by huge Reynolds numbers, indicating the turbulent character of the flows. The main successes in the description of cosmic dynamo have been achieved in the mean field dynamo theory whose development began four decades ago by Steenbek et al. [1]. The equations of the mean field dynamo are derived by applying the Reynolds approach to the MHD equations and, in the simplest case of homogeneous isotropic (mirror asymmetric) turbulence, reduce to the form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \alpha(\nabla \times \mathbf{B}) + (\eta + \beta)\Delta\mathbf{B}, \qquad (1)$$
$$\nabla \cdot \mathbf{B} = 0.$$

Here, **U** and **B** describe the averaged (large scale) fields of the velocity and magnetic field induction, $\eta = 1/\sigma\mu$ is the magnetic diffusivity coefficient (σ is the electric conductivity and μ is the magnetic permeability of the medium), α and β are the turbulent transfer coefficients describing the effect of the small-scale turbulent pulsations on the dynamics of mean fields [2, 3], α describes the generation effects (the alpha effect is the basis of most models of the mean field dynamo), and β is the contribution of turbulence to the diffusion of the large-scale magnetic field. In the kinematic approximation, the velocity field, α, and β are specified and, under certain conditions, lead to the appearance of an increasing solution of Eq. (1), i.e., the dynamo effect.

The current situation in the mean field dynamo seems to be paradoxical: during several decades, Eqs. (1) are used to develop models of MHD dynamo in various cosmic systems, although even no attempts have been made to perform direct measurements of turbulent transport coefficients for any real flows. It is thought that the simplest manifestation of the alpha effect should be based on the action of spiral turbulence, and the estimate $\alpha \approx \tau\chi/3$ (χ is the mean flow helicity and τ is the correlation time) is used. However, the single laboratory confirmation of the turbulent alpha effect was obtained in a flow where the small-scale helicity cannot ensure it, and the effect itself is due to the joint action of the gradient of turbulent pulsations and large-scale vorticity [4]. The measurements were performed at a setup used in this work.

The beta effect seems to be simpler: it is expected that the diffusion coefficient of the magnetic field in the turbulent flow, which is inversely proportional to the conductivity of the medium, should increase by analogy with the turbulent viscosity. Although the effect is considered as almost obvious, the detection of small variations in conductivity in the turbulent flow is very difficult, and the reliable direct measurements of the effective electric conductivity of the turbulent flow of a conductive fluid are absent. The only attempt at such measurements is described in [5], but the results are doubtful both in the level of observed variations in the conductivity and in estimates of the errors of the measurements. The aim of this work is to measure the effective conductivity of a low-temperature gallium alloy

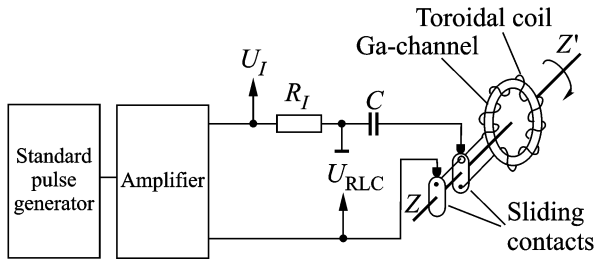

Fig. 1. Experimental layout.

under turbulent flow conditions in a closed toroidal channel.

2. EXPERIMENTAL SETUP

The experimental setup is a rigid construction with a rotating toroidal channel (the axial radius of the torus is R = 0.0875 m and the cross-section radius is r_1 = 0.0225 m) with the dielectric walls (engineering textolite) filled with a gallium alloy (87.5% Ga, 10.5% Sn, and 2% Zn) whose melting temperature is 19°C. The channel is mounted on the horizontal axis with a driving pulley, a braking disc of a car, and sliding contact rings [6, 4]. The channel rotates with a frequency up to 55 rps. The flow in the channel is generated by means of the sharp braking; the braking time is no more than 0.1 s and is reproduced for a given rotation velocity with an accuracy of no worse than 7%.

Investigations of the evolution of the nonstationary water flow in such channel show that the maximum velocity averaged over the cross section with respect to the channel wall, U, is reached at the time of the complete channel stop and is about 70% of the linear velocity of the channel axis before braking [7]. This means that the Reynolds number Re = Ur_1/ν (ν is the kinematical viscosity of the alloy) for the flow thus excited reaches the maximum value Re $\approx 10^6$, which corresponds to the magnetic Reynolds number Rm = $Ur_1/\eta \approx 1$.

The idea of the experiment reduces to the use of the dependence of the inductance L of the toroidal coil surrounding the channel with gallium on the electrical conductivity of the metal. The coil is a part of a series RLC circuit in which harmonic oscillations are excited (see Fig. 1). The expected changes in the conductivity are small (no more than a percent); hence, the phase shift between the voltage and *RLC* circuit current at the resonance frequency is chosen for the measurements.

The toroidal copper coil (∅ = 0.8 mm, N = 192 turns, and resistance $R_L = (1.60 \pm 0.07)\ \Omega$) is coiled on the channel. The current is supplied to the coil through a pair of sliding copper–graphite contacts. The current resistor $R_I = (0.5 \pm 0.025)\ \Omega$ and capacitor C of the circuit consist of thermostable components. The channel with the coil is heat insulated by expanded polystyrene and is protected from electromagnetic noise by the case made of a magnetically soft steel. The power supply of the (R_L, L, C) circuit is a low-noise audio amplifier; a sinusoidal signal from a hidh-stable pulse generator is supplied at the input of the amplifier. The voltages U_{RLC} and U_I on the circuit and current resistor, respectively, are directly measured.

The phase shift between the voltage and circuit current (up to π) coincides with the shift between the measured signals. The observed phase shifts are mainly in the range 10^{-4}–10^{-3} rad and are measured at times longer than the oscillation period by no more than one order of magnitude. For this reason, the evolution of the phase shift is calculated using the wavelet analysis and is averaged over the series.

The toroidal channel filled with the gallium alloy at a sufficient length $2\pi R \gg r_1$ can be considered as a cylindrical core of the solenoid. The magnetic field distribution over the radius $b(r)$ in the core of the solenoid of N turns of radius r_0 with alternating current I and frequency ω is given by the formula

$$b(r) = \frac{\mu_0 NI}{2\pi R}\frac{J_0(ar)}{J_0(ar_0)}, \quad a = \sqrt{\iota\omega\sigma\mu_0}, \tag{2}$$

where μ_0 is the magnetic permeability of free space and J_n is the Bessel functions.

Expression (2) allows one to calculate the magnetic flux and to obtain the following expression for the inductance of the coil with the core:

$$L = \frac{\mu_0 N^2 r_0^2}{2R}\left[1 - \left(\frac{r_1}{r_0}\right)^2 + 2\frac{r_1 J_1(ar_1)}{ar_0^2 J_0(ar_1)}\right]. \tag{3}$$

The forced electrical oscillations in the circuit are described by a second-order differential equation with the variable coefficients L and $\dot{L}$ and the a priori unknown dependence $L(t)$. However, the phase shift between the voltage and current in the quasi-static case [small variations in $L(t)$ during an oscillation period] is approximately expressed as

$$\theta \simeq \arctan\frac{\omega\Re(L) - (\omega C)^{-1}}{R_L + \omega\Im(L)}. \tag{4}$$

The real part of the inductance $\Re(L)$ determines the resonance frequency ($\theta = 0$), whereas the imaginary part $\Im(L)$ specifies the thermal losses in the channel.

Near the resonance, dependence (4) is linear in the logarithmic increments of the parameters R, r_0, r_1, and σ:

$$\begin{aligned}\delta\theta &\simeq \frac{\omega\Re(\delta L)}{R_L + \omega\Im(L)} \\ &\simeq \frac{q}{1 + qA_2/A_1}\left(-\frac{\delta R}{R} + \xi_0\frac{\delta r_0}{r_0} + \xi_1\frac{\delta r_1}{r_1} + \xi_\sigma\frac{\delta\sigma}{\sigma}\right).\end{aligned} \tag{5}$$

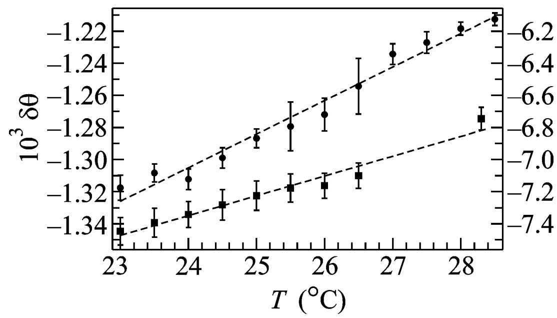

Fig. 2. Phase shift versus gallium temperature in the channel at a frequency of (upper data, left scale) 166 and (lower data, right scale) 964 Hz.

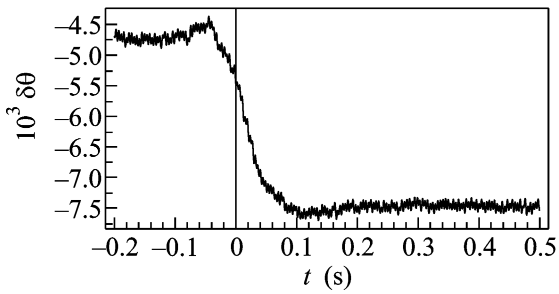

Fig. 3. Time dependence of the phase shift θ. The channel rotation frequency is 55 s^{-1}. The time $t = 0$ corresponds to the complete stop of the channel.

Here,

$$\xi_0 = 2/A_1, \quad \xi_1 = 2A_3/A_1, \quad \xi_\sigma = (1+A_3)/A_1 - 1,$$

$$q = (R_L\omega C)^{-1}, \quad A_1 = 1 - \left(1 - 2\Re\frac{J_1(z)}{zJ_0(z)}\right)\left(\frac{r_1}{r_0}\right)^2,$$

$$A_2 = 2\Im\frac{J_1(z)}{zJ_0(z)}\left(\frac{r_1}{r_0}\right)^2, \quad A_3 = \Re\left(\frac{r_1J_1(z)}{r_0J_0(z)}\right)^2,$$

$$z = (1+i)r_1/s, \quad s = (2/\mu_0\sigma\omega)^{1/2},$$

where s is the thickness of the skin layer.

All coefficients of the parameter increments are functions of the frequency ω. The increments in the experiment depend on the dynamics of the channel with the coil, flow of the gallium alloy, and on the heat release processes both in the channel (inductive heating of the alloy and heating during the flow damping) and outside it (ohmic heating of the coil). For this reason, to verify Eq. (5), we performed a test thermal experiment on the channel at rest: the $\delta\theta(T)$ dependence is determined at the stationary inductive heating of the channel. The aim of the test is to determine the temperature coefficient of the alloy resistivity, $\rho^{-1}d\rho/dT$, using this dependence and known thermal expansion coefficients of the alloy, materials of the channel wall, and coil wire.

The measurements were performed at two resonant frequencies 166.12 Hz ($C_{166} = 2420$ μF) and 963.90 Hz ($C_{964} = 88$ μF) for which the skin-layer thicknesses are about 20 and 8 mm, respectively. Figure 2 shows the measurement results. According to these results, the resistivity temperature coefficients are $(1.02 \pm 0.05) \times 10^{-3}$ K^{-1} and $(0.98 \pm 0.07) \times 10^{-3}$ K^{-1}, which are close to a tabulated value of $(1.04 \pm 0.05) \times 10^{-3}$ K^{-1} [8].

The sensitivity of the measurement system depends on the resonant frequency ω: the magnetic field at low frequencies penetrates deeper into the channel and interacts more efficiently with the turbulent flow. However, the quality factor decreases in this case. Both frequencies satisfy the quasi-stationary condition and are used in all dynamical measurements.

3. RESULTS AND CONCLUSIONS

The typical time dependence of the phase shift between the current and voltage is shown in Fig. 3. The phase shift is constant before the beginning of braking. The initial phase shift at an uniform rotation is due to the deformation of the channel by centrifugal forces, which change the geometry of the channel and coil and, as a result, the inductance of the system. The phase shift is absent when the empty channel rotates; i.e., the observed deformation is caused only by the centrifugal forces acting on the liquid metal. In subsequent measurements, the deformation-induced phase shift is subtracted according to the rotation velocity detected during the entire motion.

Figure 4 shows the time dependence of the phase shift minus the deformation contribution for various initial rotation frequencies. Each line is a result of averaging over 20 measurements. It is seen that the turbulent flow is developed in the channel with the beginning of braking; the intensity of this flow is maximal at the time of the complete stop of the channel. At this time, the phase shift is also maximal. Turbulence after the channel stop degenerates along with the mean flow and the phase shift decreases to the noise level.

Since the flow is nonstationary and the data on the evolution of the average velocity of the metal in the channel are absent, the dependence of the conductivity on the Reynolds number cannot be obtained directly from the measured curves. However, the analysis of nonstationary water flows [7] in a similar toroidal channel shows that the maximum velocity of a fluid with respect to the channel walls in this range of the Reynolds number is proportional to the initial velocity of the channel rotation. Figure 5 shows the relative change in the effective conductivity of the metal as obtained with various channel rotation frequencies from the maximum phase shift of each curve shown in Fig. 4.

The measurements show that the effective conductivity of the metal in the turbulent flow decreases with an increase in the Reynolds number (magnetic diffusivity increases), which corresponds to the general ideas on the behavior of the turbulent transfer coefficients.

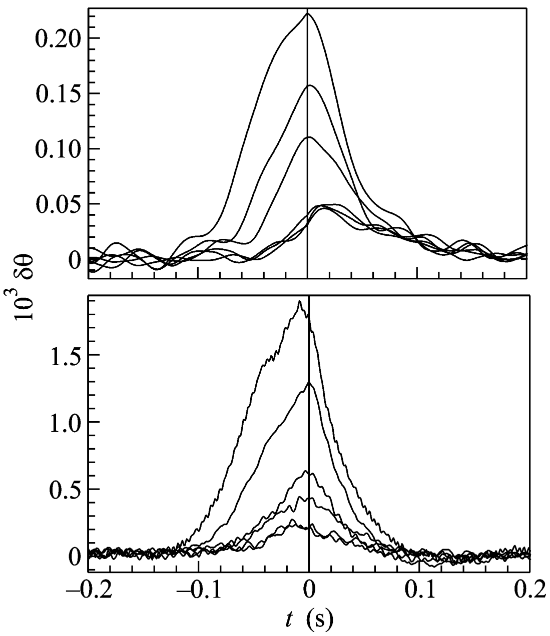

Fig. 4. Time dependence of the phase shift δθ minus the deformation contribution for the initial torus rotation frequencies (from top to bottom) 55, 50, 45, 40, and 35 s^{-1}. The resonant frequency is (upper panel) 166 and (lower panel) 964 Hz.

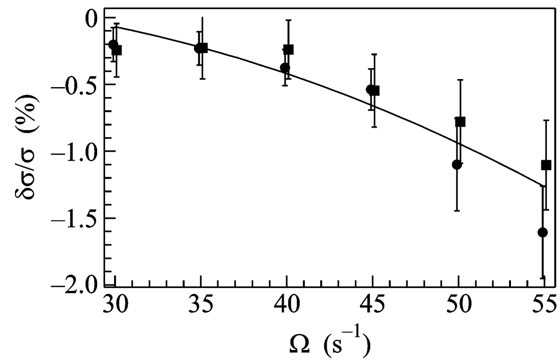

Fig. 5. Maximum relative deviation of the conductivity of the metal in the channel versus the initial channel rotation frequency Ω. The resonant frequency is (squares) 166 and (circles) 964 Hz.

According to the theory of mean field dynamo under the conditions of the uniform isotropic turbulence, the expression for the coefficient β has the general form [9]

$$\beta = \frac{4\pi}{3}\int_0^{\infty}\int_{-\infty}^{\infty}\frac{\eta k^4}{(\eta k^2)^2 + \varpi^2}W(k, \varpi)dkd\varpi. \quad (6)$$

Here, $W(k, \varpi)$ is the spectral energy density of turbulent pulsations of the velocity, k is the wavenumber, and ϖ is the frequency. When the properties of turbulence are parameterized in terms of the correlation scale $\boldsymbol{\lambda}$ and correlation time τ, Eq. (6) has two asymptotic estimates at the limiting values of the ratio of the diffusion time to the correlation time $q = \lambda^2/\eta\tau$.

In the limit of low conductivity ($q \longrightarrow 0$), we can obtain the estimate

$$\beta = (\eta/9)\mathrm{Rm}_t^2. \quad (7)$$

Here, $\mathrm{Rm}_t = u\lambda/\eta$ is the turbulent Reynolds number, where u is the rms velocity of pulsations. In the limit of high conductivity ($q \longrightarrow \infty$), we arrive at the estimate

$$\beta = \frac{\tau}{3}\langle u^2\rangle, \quad (8)$$

similar to the estimate of the turbulent viscosity $\nu_t \approx C\langle u^2\rangle^2/\varepsilon \approx C\tau\langle u^2\rangle$, where ε is the rate of the kinetic energy dissipation, which is widely used in semiempirical models of turbulence.

Estimates of form (8) do not mean that the turbulent viscosity (ν_t or β) increases as the square of the Reynolds number, because the time τ depends on the intensity and scale of turbulent pulsations. Writing $\tau \approx l/u$ and assuming that turbulence is nearly uniform and that the scale l dominates in turbulence, the $\beta(u)$ dependence becomes linear. This is not the fact in the wall flows; for example, it is well known that the turbulent viscosity in a tube at large Reynolds numbers increases as $\nu_t \sim \mathrm{Re}^{1/2}$ [10].

Note that there are other turbulent effects in addition to the effects included in Eq. (1). For example, turbulent diamagnetism, the so-called γ effect, leads to the ejection of the magnetic field from regions with a large intensity of turbulent motions [11]. However, its manifestation in this experiment can only weaken the observed effect, because it reduces the interaction of the magnetic field with the flow in the channel. Moreover, in the presence of the average motion, the turbulent electromotive force contains additional terms, which are proportional to the rotation or velocity shift and describe the anisotropic effects [12, 13]. However, their weight coefficients for this experiment are much smaller than those for isotropic effects [14]. For this reason, their contribution can be neglected. Finally, the above analysis indicates that the turbulent magnetic diffusivity is the only probable explanation of the experimental results.

Thus, in the investigated developed turbulent flow of the metal ($\mathrm{Re} \approx 10^6$), the measured changes in the magnetic diffusivity in the magnetic Reynolds number range $\mathrm{Rm} < 1$ and $q \approx 0.1$ are described by the dependence $\beta \sim (\mathrm{Rm})^2$. The exponent likely decreases with an increase in the magnetic Reynolds number, because there are no reasons to expect that the behavior of the turbulent magnetic diffusivity at $\mathrm{Rm} \longrightarrow \mathrm{Re} \gg 1$ is significantly different from the behavior of its kinematical analog. There are also no reasons to expand the resulting dependence on the turbulent characteristics of the electroconductive medium in the case of an uniform turbulence. Nevertheless, taking into account the difficulties of the measurements of the considered characteristic in real flows and the absence of the experimental data, measurements of the effective electric conductivity of the turbulent medium even for one particular case constitute a very important stage in the experimental justification of modern concepts of MHD dynamo.

Experimental investigations of the beta effect at much larger Reynolds numbers (Rm $\gg$ 1) are obviously of great interest. Such regimes will be possible after the production of a larger channel with walls made of a much more rigid material and a change of the gallium alloy to liquid sodium, which is a much lighter and well-conductive metal.

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